Journal of Forestry Research and Management. Vol. 15(1).186-196; 2018, ISSN 0189-8418
www.frin.gov.ng/frin1/journals.html

EVALUATION OF PARAMETER ESTIMATORS FOR FITTING WEIBULL DISTRIBUTION TO CROWN AREA OF *Parkia biglobosa* Jacq. and *Prosopis africana* Guill & Perr. IN MAKURDI, NIGERIA

Chukwu O.[1]*, J. H. Dau[2]and J. U. Ezenwenyi[3]

[1]Department of Social and Environmental Forestry, University of Ibadan, Ibadan, Nigeria

[2]Department of Forest Production and Products, University of Agriculture, Makurdi, Nigeria

[3]Department of Forestry and Wildlife, Nnamdi Azikiwe University Awka, Nigeria

*Corresponding Author: E-mail: onye20042000@yahoo.com; Phone No: +234(0)8032633835

ABSTRACT

The validity and appropriateness of any distribution function in fitting a data set depends on its parameter estimators. Maximum likelihood and percentile estimators were used in fitting three-parameter (3P) Weibull distribution to crown area of economic tree species found in a natural forest within the Federal University of Agriculture, Makurdi, Nigeria. Diameter at breast height (Dbh) and crown diameter were measured from all living *Parkia biglobosa* and *Prosopis africana* trees with Dbh ≥10.0 cm found within forty (40) temporary sample plots of 50 m x 50 m size; established using systematic random sampling method. The two estimators were evaluated based on mean values of Bias, mean absolute error (MAE), mean square error (MSE), and Kolmogorov Smirnov (K-S) test. The study revealed that maximum likelihood estimator was more stable in its parameter estimation with least MAE (0.5044), MSE (3.2745) and K-S (0.0551). Hence, it was recommended for fitting Weibull (3P) distribution to crown area of *Parkia biglobosa* and *Prosopis africana* in the study area for sustainable management.

Keyword: Crown area, Maximum likelihood, Percentile estimator, Weibull distribution

INTRODUCTION

Forests are large areas of land which have great variety of trees, shrubs and herbs that provide timber and non-timber benefits to humanity, particularly the rural people whose livelihoods are closely interwoven around the plants available in their domain. Such plants provide them with food, medicine, gum, resins, construction materials, nuts and seeds, tools, decorations, cosmetics, cultural and artifacts and other needs (Oboho and Ogana, 2011). However, tree crown plays an essential role in tree productivity in that it is the location of the physiological activity that leads to growth and development. Distribution of crown dimensions affects light penetration and absorption during photosynthesis (Monsi and Saeki, 2005).

Tree crown is major component of wildlife habitat and understory light conditions for regeneration (MacArthur and MacArthur, 1961; Crookston and Stage, 1999). Crown variables have become integral in remote sensing especially in fields using LiDAR (Light Detection and Ranging), MODIS (Moderate-resolution imaging spectroradiometer) and airborne laser scanning technologies (Woodward *et al*., 2004; Salas *et al*., 2010). It is therefore, necessary to understand the distribution pattern of tree crown as it is the anchor for fruits, leafs and branches.

Parkia biglobosa and *Prosopis africana are* common economic tress found around villages in the Savannah areas of West Africa, where they are left standing when land is cleared or sometimes planted and trees are individually owned (Hutchinson and Dalziel, 1963). They are dicotyledonous angiosperm, belonging to the family *Fabaceae* (Alabi *et al.*, 2005). The plants are categorized under vascular plants; they are deciduous perennial plants that grow up to between 7 and 20 m height, in some cases up to 30 meters (Ntui *et al.*, 2012). *P. biglobosa* species is commonly referred to as "Daddawa" in Hausa, "Ogiri" in Igbo and "Iru" in Yoruba languages of Nigeria. Also, *P. africana* is locally called iron wood (English), Okpei (Igbo), Ayan (Yoruba), Okpeghe (Idoma and Tiv), and Kiriya (Hausa). Both species have several uses, including fodder, food, medicine, green manure, fuel wood, timber and economic purposes (Sabiiti and Cobbina, 2012). The two tree species were selected based on their economic values to Northern and South-western parts of Nigeria.

Weibull function has been the most widely used distribution function for describing continuous variables; because of its flexibility and relative simplicity. It has been successfully used in forestry for fitting tree stem diameter and basal area (Ige *et al.*, 2013; Ogana and Gorgoso-Varela, 2015). Little have been done with regards to crown dimensions. Studies (Datta and Datta, 2013; Ogana and Gorgoso-Varela, 2015) have shown that despite the simplicity and flexibility of Weibull function, the estimation of its parameters can be difficult. Hence, the validity and appropriateness of its result depends on the parameter estimator used. Therefore, a successful crown area distribution model requires good prediction of its parameters.

Previous studies (Ogana and Gorgoso-Varela, 2015) on probability density functions have shown that the number of parameter of a function affects its performance. Ogana and Gorgoso-Varela (2015) attributed the outstanding performance of Weibull (3P) over Weibull (2P) to the number of their parameter, as 2-parameter Weibull distribution provided a poor fit to data compared to that of the 3-parameter Weibull. Hence, this study adopted 3-parameter Weibull function for distribution of crown areas of *Parkia biglobosa* and *Prosopis africana* species found in the natural forest of the Federal University of Agriculture, Makurdi, Nigeria using maximum likelihood and percentile parameter estimators; and further compared the effectiveness of the two estimators in fitting the Weibull (3P) function for sustainable management of the economic tree species investigated in the study area.

MATERIAL AND METHOD

Study Area

This study was carried out in at the Federal University of Agriculture Makurdi, Benue State, Nigeria. It is located on latitude 7° 33'N and 7°52'N and longitude 8° 21'E and 8°39'E and in Benue State, within the southern guinea savannah ecological zone of Nigeria and covers a total land area of 7,978 km^2. The topography of the study area is characterized by gentle hills. The soil is mainly sandy-loamy; the climate is characterized by distinct rainy and dry seasons. The annual rainfall ranges between 1016 mm to 1524 mm spreading over May to October (Gyang, 1997).

Sampling Procedure and Data Collection

Data used in this study was collected from forty (40) selected Temporal Sample Plots (TSPs) of size 50 m × 50 m (0.25 ha) within the forest in study area of about 19.3 ha size using systematic random sampling. Within each TSP, the following tree growth variables of living *P. biglobosa* and *P. africana* trees with diameter at breast height (Dbh) ≥10.0 cm in the selected plots were measured; diameter at breast height (cm) and crown diameter (m).

Crown Area (CA) was further computed as;

$$CA = \frac{\pi CD^2}{4} \quad (1)$$

Where, CD = crown diameter (m) and π = Pi (3.146)

DATA ANALYSIS

Fitting Procedure

Three-parameter Weibull distribution (Weibull 1951) was used in this study. It is expressed as:

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (2)$$

The Weibull cumulative distribution function (CDF) was obtained by the integration of the above function. It is expressed as:

$$F(x) = \int_0^x \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] dx \quad (3)$$

$$F(x) = 1 - exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (4)$$

Where: *F(x)* is the Weibull cumulative distribution function; *x* is tree diameter measured, a, b and c are the location, scale and shape parameters of the distribution respectively.

Parameters estimation of Weibull function

In this study, two methods were used to estimate the parameters of the Weibull distribution. These include: Percentiles (PW) and Maximum Likelihood (MLW).

1. *Percentiles Method (PW)*

The percentile method below was used to estimate the parameters of the Weibull distribution as applied by Zanakis (1979). The values of the parameters were computed with the following expressions:

$$\hat{a} = \frac{X_1 X_n - X_2^2}{X_1 + X_n - 2X_2} \quad (5)$$

If X_2 is closer to X_1 than to X_n, and X_1 otherwise

$$\hat{b} = -\hat{a} + X_{(0.63_n)} \quad (6)$$

$$\hat{c} = \frac{\ln\left[\frac{\ln(1-p_k)}{\ln(1-p_i)}\right]}{\ln\left[\frac{X_{(np_k)} - \hat{a}}{X_{(np_i)} - \hat{a}}\right]} \quad (7)$$

Where; P_i and P_k are constants with proposed values by Zanakis (1979):0.16731 and 0.97366 respectively, Xi= the i^{th} ordered value (from smallest to largest) in the sample and n = sample size.

Maximum Likelihood (MLW)

This method involves taking the partial derivatives of the log-likelihood function with respect to each of the distributions' parameters and setting the expression equal to zero and then, solve by a numerical iterative algorithm such as the Newton-Raphson approach to yield the ML estimates. The Likelihood Function for the Weibull distribution is given by:

$$L(\theta) = \prod_{i=1}^{n} \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (8)$$

$$LogL(\theta) = nLog(c) - ncLog(b) + (c-1)\sum_{i=1}^{n} log(x_i - a) - \sum_{i=1}^{n}\left(\frac{x_i - a}{b}\right)^{c} \quad (9)$$

Where: $L(\theta)$ = likelihood for distribution; n = number of trees; parameters are defined as before

Model Comparison

The bias, mean absolute error (MAE), mean square error (MSE) and Kolmogorov Smirnov (K-S) goodness of fit measures was used in this study. They were computed for each fit in mean relative frequency of trees per one for all crown area classes. Hence, method with the least values of these measures was selected. They were computed as:

$$Bias = \frac{\sum_{i=1}^{N} Y_i - \hat{Y}_i}{N} \quad (10)$$

$$MAE = \frac{\sum_{i=1}^{N} |Y_i - \hat{Y}_i|}{N} \quad (11)$$

$$MSE = \frac{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}{N} \quad (12)$$

$$K - S\ (D_n) = Supx|F(x_i) - F_0\ (x_i)| \quad (13)$$

Where: Supx is the supremum value, $F(x_i)$ is the cumulative frequency distribution observed for the sample x_i (i = 1, 2…, n) $F_0(x_i)$ is the probability of the theoretical cumulative frequency distribution. CA classes of 1cm intervals were selected.

RESULTS

The summary statistics for the tree growth variables measured from the study area (Table 1.) showed that for *P. biglobosa* the distribution of diameter at breast height (Dbh) ranged from 25.13cm to 86.75 cm, crown diameter (CD) ranged from 2.90 m to 11.55 m and crown area (CA) ranged from 6.61m^2 to 104.77 m^2. However, for *P. africana* the distribution of diameter at breast height (Dbh) ranged from 17.27 cm to 63.80 cm, crown diameter (CD) ranged from 3.10 m to 10.80 m and crown area (CA) ranged from 7.07m^2 to 91.69 m^2. The mean values with corresponding standard error (SE) for the growth variables were presented in the Table 1.

A total number of four hundred and ninety-two (492) trees were measured in all the forty selected plots (*P. biglobosa* (265 trees) and *P. africana* (227 trees)).

Journal of Forestry Research and Management. Vol. 15(1).186-196; 2018, ISSN 0189-8418
www.frin.gov.ng/frin1/journals.html

Table 1: Summary statistics of the data from the Study Area

Species	Growth variable	N	Descriptive Statistics		
			Minimum	Maximum	Mean± SE
Parkia biglobosa	Dbh	265	25.13	86.75	38.8887 ± 0.7553
	CD	265	2.90	11.55	6.4055 ± 0.1141
	CA	265	6.61	104.77	34.9229 ± 1.2428
Prosopis africana	Dbh	227	17.27	63.80	30.8302 ± 0.5822
	CD	227	3.10	10.80	6.2037 ± 0.1069
	CA	227	7.07	91.69	32.2551 ± 1.1308

Dbh = Diameter at breast height (cm), CD = Crown diameter (m), CA = Crown area (m^2), N= number of trees and SE = Standard error

The comparison of maximum likelihood and percentile methods for fitting the 3-parameter Weibull distribution to the crown area from the study area was made and the results are shown in Figure 1. Graphical analyses of the observed numbers of trees and the predicted frequency by Weibull distribution was no doubt typical of a natural forest, where a larger proportion of tree crowns were found in the smaller diameter classes with decreasing frequency as the diameter increases. The graph also revealed that, the expected frequency of trees produced by 3-parameter Weibull distribution fitted with maximum likelihood and percentile methods exhibited slight variation with the observed diameter distribution. The two fitting methods predicted lesser values than the observed distribution for the larger diameter classes of 20.5, 24.5, 31.5-38.5 cm and 48.5-50.5 cm.

Journal of Forestry Research and Management. Vol. 15(1).186-196; 2018, ISSN 0189-8418
www.frin.gov.ng/frin1/journals.html

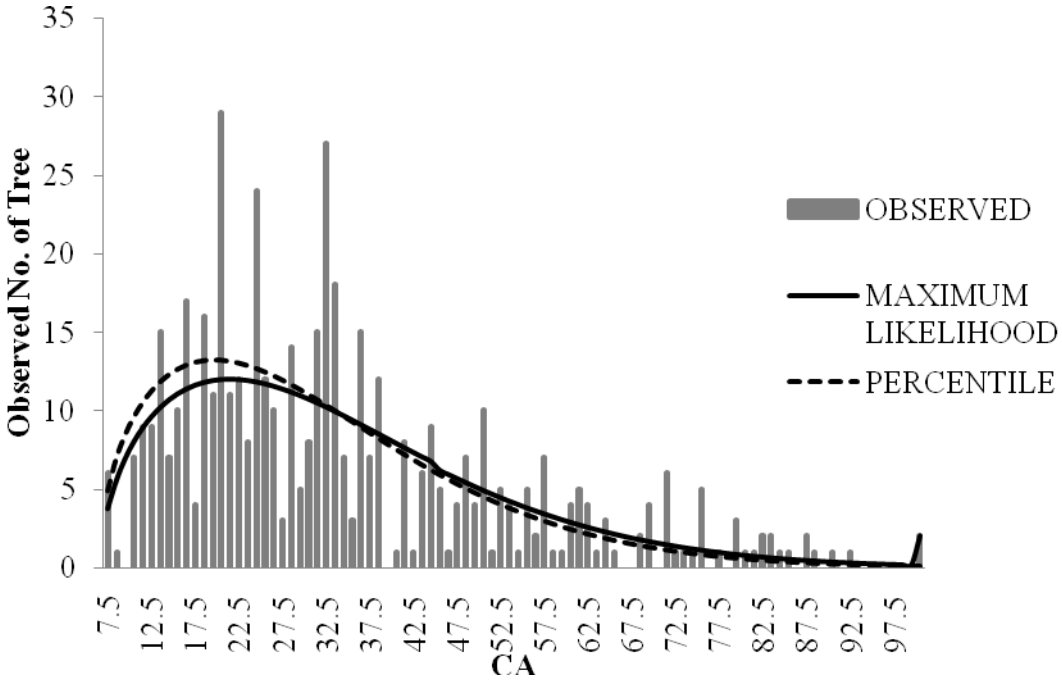

Figure 1: Observed CA fitted with 3P Weibull distribution using maximum likelihood and percentile estimators.

The result of the goodness of fit (Table 2) showed that Maximum likelihood had the least values of MAE, MSE and K-S; 0.5044, 3.2745 and 0.0551 respectively while percentile had 0.5263 of MAE, 3.3139 of MSE and 0.0604 value of K-S. Furthermore, the result also indicates that percentile had the least mean value of Bias (0.0034) as against maximum likelihood (0.0132). as shown in below. Figures 2-4, showed the graphical patterns of comparison criteria (Bias, MAE and MSE) of the two estimation methods.

Table 2: Goodness of fit for 3P Weibull distribution

Fitting Method	Bias	MAE	MSE	K-S
Maximum likelihood	0.0132	0.5044	3.2745	0.0551
Percentile	0.0034	0.5263	3.3139	0.0604

Where: MAE= mean absolute error, MSE= mean square error and K-S= Kolmogorov Smirnov

DISCUSSION

Tree Crown distribution is an effective method for describing a forest stand because tree productivity, gaseous exchange and health depends largely on the crown dimensions (Zarnoch *et al.*, 2004; Adesoye and Ezenwenyi, 2015). In forestry, the Weibull distribution function has been widely used in describing diameter prior to its flexibility and relative simplicity. Hence, a successful crown area distribution model requires good prediction of its parameters.

In this study mean values of Bias, mean absolute error (MAE), mean square error (MSE) and Kolmogorov Smirnov (K-S) statistic were used to assess/ adjudge the fit performance of the maximum likelihood and percentile estimators in fitting the Weibull distribution to crown area. The result of the goodness of fit shows that maximum likelihood estimator was more consistent than percentile and as such chosen as the best for the study area. This study is in congruent with the work of Zhang *et al.*, (2003) who obtained better results with maximum likelihood method than percentile for fitting the 3-parameter Weibull distribution. In the same vein, the study agrees with the works of Ogana and Gorgoso-Varela (2015) that fitted Weibull function to bole diameters of tree species in natural forest of Nigeria.

Thus, the study also revealed that percentile had smallest mean value of Bias, which disagrees with the work of Zarnoch and Dell (1985), that maximum likelihood estimators had superior smaller bias than Percentile estimation method for Weibull 3-parameters in 20 loblolly pine plots. Zarnoch and Dell (1985) also stated that, percentile estimators, however, should not be dismissed as totally unsuitable. Their simplicity is a valuable attribute and their performance when c (location) parameter is near or below 2 is comparable to or better than that of the maximum likelihood.

CONCLUSIONS

Percentile and maximum likelihood methods can be used to estimate 3-parameter Weibull distribution for crown area of *P. biglobosa* and *P. africana* in Makurdi, Nigeria. Maximum likelihood gave an overall better estimation and therefore is recommend for estimating Weibull

parameters (3P) and fitting the Weibull distribution to the crown area of *P. biglobosa* and *P. africana* in the study area.

Furthermore, tree crown dimensions have a high value in forest ecosystem management, where the forest structure, wildlife habitat suitability, fruit and seed production or shading potentials is considered a high priority. Hence, it is suggested that other distribution functions and parameter estimators should be employed in exploring crown dimensions.

REFERENCES

Adesoye, P.O. and Ezenwenyi, J.U. (2014). Crown Diameter Prediction Models for *Tectona Grandis* Linn.f. in Omo Forest Reserve, Nigeria, *Journal of Forestry Research and Management. Vol.11, 72-87; 2014. ISSN: 0189-8418*

Alabi, D.A., Akinsulire, O.R. and Sanyaolu, M.A. (2005). *African Journal of Biotechnol*; Vol. 4; Pp. 812-815.

Crookston, N.L. and Stage, A.R. (1999). Percent Canopy Cover and Stand Structure Statistics from the Forest Vegetation Simulator.General Technical Report RMRS-GTR-24. 34p

Datta, D. and Datta D. (2013). Comparison of Weibull Distribution and Exponentiated Weibull Distribution Based Estimation of Mean and Variance of Wind Data, *International Journal of Energy, Information and Communications.* Vol. 4, Issue 4.

Gyang, E.O. (1997). Students' information handbook (1996-1998). University of Agriculture, Makurdi. P. 1-3.

Hutchinson, J. and Dalziel, J.M. (1963). Flora of West Tropical Africa.Crown Agents, London, 6 vols.

Ige, P.O., Akinyemi, G.O. and Abi, E.A. (2013). Diameter Distribution Models for Tropical Natural Forest trees inOnigambari Forest Reserve, *Journal of Natural Sciences Research*, 3(12):14-22.

MacArthur, R.H. and MacArthur, J.W. (1961). On bird species diversity. Ecology 42, 594–598.

Monsi, M. and Saeki, T. (2005). On the factor light in plant communities and its importance for matter production (English translation). *Annals of Botany*. 95, 549–567.

Ntui, V.O., Uyoh, E A., Urua, I S., Ogbu, U., and Okpako, E.C. (2012). Regeneration of *Parkia biglobosa Benth*.: An important tree species of Africa. *Journal of Microbiology and Biotechnology Research*, 2(1), 169-177;

Oboho, E.G. and Ogana, F.N. (2011). Effects of Varying Water Soaking Duration On The Germination Of *Garcinia Kola* (Heckel) Seeds. *Nigerian Journal of Agriculture, Food and Environment.* 7(2):57-62

Ogana, F.N. and Gorgoso-Varela, J.J. (2015). Comparison of Estimation Methods for Fitting Weibull Distribution to the Natural Stand of Oluwa Forest Reserve, Ondo State, Nigeria. *Journal of Research in Forestry, Wildlife and Environment*. 7(2): 81-90

Salas C., Ene, L., Gregoire, T.G., Næsset, E. and Gobakken, T. (2010). Modelling tree diameter from airborne laser scanning derived variables: A comparison of spatial statistical models, *Remote Sensing of Environment*, 114:1277–1285.

Sabiiti, E.N. and Cobbina, J. (2012). *Parkia Biglobosa:* A Potential Multipurpose Fodder Tree Legume in West Africa. *International Tree Crops Journal*. 7, 1992 (3): 113-139.

Weibull, W. (1951). A Statistical distribution function of wide applicability. *J. Appl. Mech*. 18: 293-297

Woodward, F.I., Lomas, M.R. and Kelly, C.K. (2004). Global climate and the distribution of plant biomes.*Philos. Trans. R. Soc. B* 359:1465–76

Zanakis, S.H. (1979). A simulation study of some simple estimators for the three-parameter Weibull distribution. *Journal of Statistical Computation and Simulation*, 9:101-116.

Zarnoch, S.J. and Dell, T.R. (1985). An Evaluation of percentile and maximum likelihood estimators of Weibull parameters, *Forest Sci.,* The Society of American Foresters. 31(1): 260-268

Zarnoch, S.J., Bechtold, W.A. and Stolke, K.W. (2004). Usingcrown condition variables as an indicator of forest health. *Canadian Journal of Forestry Resource*. 34:1057-1070.

Zhang, L., Kevin, C.P. and Chuanmin, L. (2003). A comparison of estimation methods for fitting Weibull and Johnson's SB distributions to mixed spruce-fir stands in north-eastern North America. *Canadian Journal of Forestry Resource*. 33: 1340-1347.